

2021 八省适应性考试(数学)答案

一、选择题

BCACBDBD

二、多选题

AC BC BD AD

三、填空题

13. $61\pi i$

14. $\frac{1}{3}, -3$

15. $\sin \pi x$

16. 32

四、解答题

17.

(1) $a_{n+2} + a_{n+1} = 3(a_{n+1} + a_n)$, 又 $a_2 + a_1 \neq 0$, 则数列 $\{a_{n+1} + a_n\}$ 为等比数列.

(2) $a_{n+1} + a_n = 2 \cdot 3^{n-1}$, 则 $a_{n+1} - 3a_n = -(a_n - 3a_{n-1}) = (a_2 - 3a_1) \cdot (-1)^{n-1} = 0$, $a_{n+1} = 3a_n$, $a_n = \frac{1}{2} \cdot 3^{n-1}$.

18.

(1) $BC = \frac{\sqrt{2}}{2}$

(2) $\cos \angle BDC = \sqrt{3} - 1$

19.

(1) $P = 1 - 0.9 \times 0.8 = 0.18$

(2)

X	0	1	2	3
P	0.504	0.398	0.092	0.006

$E(X) = 0.6$

20.

(1) $2\pi \cdot 4 - 6\pi = 2\pi$

(2) 设多面体有 a 个顶点, b 条棱, c 个面, 各个面分别是 k_1, k_2, \dots, k_c 边型,

则面角和为 $\pi(k_1 - 2) + \pi(k_2 - 2) + \dots + \pi(k_c - 2)$,

又 $k_1 + k_2 + \dots + k_c = 2b$,

则总曲率为 $2\pi a - \pi(k_1 + k_2 + \dots + k_c - 2c) = 2\pi a - \pi(2b - 2c) = 2\pi(a - b + c) = 4\pi$

21.

(1) $|BF| = |AF|$, $\frac{b^2}{a} = a + c$, 则 $e = \frac{c}{a} = 2$.

(2) 设 $B(x_0, y_0)$, $\tan \angle BFA = \frac{y_0}{c - x_0}$, $\tan \angle BAF = \frac{y_0}{x_0 + a}$

题设等价于证明 $\tan \angle BFA = \tan 2\angle BAF$, 即 $\frac{y_0}{c - x_0} = \frac{\frac{2y_0}{x_0 + a}}{1 - \left(\frac{y_0}{x_0 + a}\right)^2} = \frac{2y_0(x_0 + a)}{(x_0 + a)^2 - y_0^2}$,

由于 $e = \frac{c}{a} = 2$, $y_0^2 = 3(x_0^2 - a^2)$,

则 $\frac{2y_0(x_0 + a)}{(x_0 + a)^2 - y_0^2} = \frac{2y_0(x_0 + a)}{(x_0 + a)^2 - 3(x_0^2 - a^2)} = \frac{2y_0}{(x_0 + a) - 3(x_0 - a)} = \frac{2y_0}{4a - 2x_0} = \frac{y_0}{2a - x_0} = \frac{y_0}{c - x_0}$,

即证.

22.

(1) 注意到 $f(0) = 0$,

当 $x \in \left(-\frac{5\pi}{4}, -\frac{\pi}{4}\right]$ 时, $f(x) = e^x - \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) > 0$;

当 $x \in [0, +\infty)$ 时 $f(x) = e^x - \sin x - \cos x \geq e^x - x - 1 \geq 0$;

当 $x \in \left(-\frac{\pi}{4}, 0\right)$ 时, $f'(x) = e^x - \cos x + \sin x$, $f'(0) = 0$,

$f''(x) = e^x + \sin x + \cos x = e^x + \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) > 0$,

则函数 $f(x)$ 在 $\left(-\frac{\pi}{4}, 0\right)$ 上单调增, 则 $f(x) \leq f(0) = 0$,

则函数 $f(x)$ 在 $\left(-\frac{\pi}{4}, 0\right)$ 上单调减, 则 $f(x) \geq f(0) = 0$;

综上所述即证.

(2) $a = 2$.